Exercise 6

The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm?

Solution

The volume of a sphere with radius r is

$$V = \frac{4}{3}\pi r^3.$$

Differentiate both sides with respect to t, using the chain rule on the right side.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = (4\pi r^2) \cdot \frac{dr}{dt}$$

The radius is increasing by 4 millimeters per second, so $dr/dt = 4$ mm/s. Therefore, when the diameter is 80 mm (that is, when $r = 40$), the rate that the volume is increasing is

$$\left.\frac{dV}{dt}\right|_{r=40} = 4\pi(40)^2(4) = 25\,600\pi\ \frac{\text{mm}^3}{\text{s}}.$$